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NEW SCHEME

First Semester B.E. Degree Examination, Dec. 06 / Jan. 07
Common to All Branches
Engineering Mathematics – I

Time: 3 hrs.] [Max. Marks:100

Note: ***Attempt any FIVE full questions choosing atleast TWO questions from each part.***

PART A

1 a. If $y = \log_{10}\left[(1-2x)^3(8x+1)^5\right]$ find y_n. **(07 Marks)**

b. If $y = \log\left(x+\sqrt{1+x^2}\right)$ show that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2 y_n = 0$. **(07 Marks)**

c. Find the pedal equation of the curve $r = ae^{m\theta}$. **(06 Marks)**

2 a. State and prove Euler's theorem for f(x, y), a homogenous function of degree n, and prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)f(x,y)$. **(07 Marks)**

b. If u = x log (xy) where $x^3 + y^3 + 3xy = 1$, find $\frac{dy}{dx}$ and hence find $\frac{du}{dx}$. **(07 Marks)**

c. If $u = x^2 - y^2$, $v = 2xy$ and $x = r\cos\theta$, $y = r\sin\theta$, determine the value of the Jacobian $\frac{\partial(u,v)}{\partial(r,\theta)}$. **(06 Marks)**

3 a. Using the reduction formula, evaluate $\int \tan^6 x\,dx$. **(07 Marks)**

b. If n is a positive integer, show that $\int_0^{2a} x^n\sqrt{2ax-x^2}\,dx = \frac{(2n+1)!}{(n+2)!\,n!}\frac{a^{n+2}}{2^n}\pi$. **(07 Marks)**

c. Trace the curve $r^2 = a^2\cos 2\theta$. **(06 Marks)**

4 a. If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$, find $\frac{ds}{d\theta}$. **(07 Marks)**

b. Find the area between the curve $x^2y^2 = a^2(y^2 - x^2)$ and its asymptotes $x = \pm a$. **(07 Marks)**

c. By differentiation under integral sign, show that $\int_0^{\pi}\frac{\log(1+a\cos x)}{\cos x}dx = \pi\sin^{-1}a$. **(06 Marks)**

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PART B

5 a. Solve $\frac{dy}{dx} = (4x+y+1)^2$. **(07 Marks)**

b. Solve $y' = \frac{xy^2-1}{1-x^2y}$. **(07 Marks)**

c. Find the orthogonal trajectories of the family of circles $x^2 + y^2 = 2cx$. **(06 Marks)**

6 a. Discuss the nature of the series:

$\frac{x}{1}+\frac{1}{2}.\frac{x^3}{3}+\frac{1.3}{2.4}\frac{x^5}{5}+\frac{1.3.5}{2.4.6}\frac{x^7}{7}+ - - - - - - - - - - \infty$. $x>0$ **(07 Marks)**

b. Find the nature of the series:

$\frac{3}{4}x+\left(\frac{4}{5}\right)^2 x^2+\left(\frac{5}{6}\right)^3 x^3 + - - - - - - - - - \infty$. $x>0$ **(07 Marks)**

c. Test the series $1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+ - - - - - - \infty$ for absolute convergence. **(06 Marks)**

7 a. Find the equation of the line drawn through the point (1, 0, -1) and intersecting the lines $x = 2y = 2z$ and $3x + 4y = 1$, $4x + 5z = 2$. **(07 Marks)**

b. Find the equations of the two planes which bisect the angles between the planes $3x - 4y + 5z = 3$, $5x + 3y - 4z = 9$. Also point out which of the planes bisect the acute angle. **(07 Marks)**

c. Find the magnitude and the equations of the shortest distance between the lines $\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}$ and $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$. **(06 Marks)**

8 a. Find the tangential and normal components of acceleration of a particle moving along curve $x(t) = t^2$, $y(t) = -t^3$, $z(t) = t^4$ at $t=1$. **(07 Marks)**

b. If $\vec{F} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$ find $\text{div}\vec{F}$ and $\text{curl}\vec{F}$ at (1, 2, 3). **(07 Marks)**

c. Prove that curl $(\text{grad}\varphi) = 0$. **(06 Marks)**

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First Semester B.E. Degree Examination, July 2007
Common to All Branches
Engineering Mathematics – I

Time: 3 hrs.] [Max. Marks:100

Note : ***Answer any FIVE full questions choosing atleast two from each part.***

PART A

1 a. Find the n^{th} derivative of $y = x^2 Cos^2(3x)$. **(07 Marks)**

b. If $y^{1/m} + y^{-1/m} = 2x$, find the value of $(x^2-1)y_{n+2} + (2n-1)xy_{n+1}$ using Leibnitz's theorem. **(07 Marks)**

c. Find the pedal equation of curve $r^2 = a^2 Sec(2\theta)$. **(06 Marks)**

2 a. If f(X, Y) is a homogeneous function of degree 'n' then prove that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$ and $x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2} = n(n-1)f$. **(07 Marks)**

b. If $X = e^V sec(u)$, $Y = e^V tan(u)$ prove that $JJ' = 1$. **(07 Marks)**

c. The time 't' of oscillation of a simple pendulum of length 'l' is given by $t = 2\pi\sqrt{\frac{l}{g}}$, where 'g' is constant. What is the approximate error in the calculated value of 't' corresponding to error of 2% in the value of 'l'? **(06 Marks)**

3 a. Evaluate $\int Sin^5 x dx$ using reduction formula and hence find $\int_0^{\pi/2} Sin^5(x)dx$. **(07 Marks)**

b. Evaluate $\int_0^{\pi/6} Cos^4(3x) Sin^2(6x)dx$ using reduction formula. **(07 Marks)**

c. Trace the curve $xy^2 = a^2(a-x)$. **(06 Marks)**

PART B

4 a. Find $\frac{ds}{d\theta}$ and $\frac{ds}{dr}$ for the curve $r = a(1 + Cos\theta)$. **(07 Marks)**

b. Find the volume of the solid generated by revolving the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ about x-axis. **(07 Marks)**

c. Using differentiation under integral sign, evaluate $\int_0^{\infty} \frac{e^{-x}}{x}(1 - e^{-\alpha x})dx$, $\alpha > -1$. **(06 Marks)**

Contd.... 2

5 a. Solve: $\frac{dy}{dx} = \frac{y^2 + y + 1}{x^2 + x + 1}$ (05 Marks)

b. Solve: $xdy - ydx = \sqrt{x^2 + y^2}\,dx$. (05 Marks)

c. Solve: $(2x + y - 1)dy = (x - 2y + 5)dx$ (05 Marks)

d. Find orthogonal trajectories of family of cardioides $r = a(1 - \cos\phi)$. (05 Marks)

6 a. State: i) Comparison test ii) Ratio test iii) Cauchy's root test. (07 Marks)

b. Show that the series $\sum \frac{1}{n^P}$ converges if P>1 and diverges if $P \leq 1$. (07 Marks)

c. Test the convergence of the series $\sum \frac{[(n+1)x]^n}{n^{n+1}}$. (06 Marks)

7 a. Find the angle between the lines whose direction cosines satisfy the relations $l + 3m + 5n = 0$ and $2mn - 6nl - 5lm = 0$. (07 Marks)

b. Find the length and the foot of the perpendicular dropped from the point (3, 2, 1) onto the plane passing through the points (1, 1, 0), (3, -1, 1) and (-1, 0, 2). (07 Marks)

c. Find the shortest distance between the lines $x + 2y - 3z - 2 = 0;\ 2x - y - z + 1 = 0$ and $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z}{3}$. (06 Marks)

8 a. Find the tangential and normal components of acceleration of a particle moving along curve $x(t) = t^2$, $y(t) = -t^3$, $z(t) = t^4$ at $t = 1$. (07 Marks)

b. If $\vec{F} = \text{grad}(x^3y + y^3z + z^3x - x^2y^2z^2)$, find div. $\vec{F}$ and curl $\vec{F}$ at (1, 2, 3). (07 Marks)

c. Prove that curl(gradΦ) = 0. (06 Marks)

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06MAT11

First Semester B.E. Degree Examination, Dec. 07 / Jan. 08
Engineering Mathematics I

Time: 3 hrs. Max. Marks:100

Note : *Answer any FIVE full questions choosing at least two questions from each part.*

Part A

1 a. Find the n^{th} derivatives of,
   i) $e^{-x}\sin^2 x$.
   ii) $\dfrac{x}{(x-1)(2x+3)}$ **(07 Marks)**

b. Prove that
$$D^n\left[\frac{\log x}{x}\right]=\frac{(-1)^n n!}{x^{n+1}}\left[\log x-1-\frac{1}{2}-\frac{1}{3}\cdots\cdots-\frac{1}{n}\right].$$ **(07 Marks)**

c. With the usual notation, prove that
$$\frac{1}{P^2}=\frac{1}{r^2}+\frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2.$$ **(06 Marks)**

2 a. If $u=\sin^{-1}\left(\dfrac{3x^2+4y^2}{3x+4y}\right)$, prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=\tan u$. **(07 Marks)**

b. If $u=f(x-y,\, y-z,\, z-x)$, prove that $\dfrac{\partial u}{\partial x}+\dfrac{\partial u}{\partial y}+\dfrac{\partial u}{\partial z}=0$. **(07 Marks)**

c. If $x=e^u\cos v$ and $y=e^u\sin v$, show that $J\cdot J'=1$. **(06 Marks)**

3 a. Obtain the reduction formula for $I_n=\int_0^{\pi/2}\cos^n x\,dx$, where n is a positive integer and hence evaluate I_5. **(07 Marks)**

b. Evaluate : $\int_0^{2a} x^2\sqrt{2ax-x^2}\cdot dx$. **(07 Marks)**

c. Trace the curve $y^2(a-x)=x^3$, where $a>0$. **(06 Marks)**

4 a. For the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$, find $\dfrac{ds}{dx}$ and $\dfrac{ds}{dy}$. **(07 Marks)**

b. Find the area of the cardioid $r=a(1+\cos\theta)$. **(07 Marks)**

c. By the differentiation under integral sign, evaluate $\int_0^1 \dfrac{x^\alpha-1}{\log x}dx$, given $\alpha\ge 0$. **(06 Marks)**

Part B

5 a. Solve :

i) $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$.

ii) $(1+y^2)dx = \left(\tan^{-1} y - x\right)dy$.

iii) $\left(5x^4 + 3x^2y^2 - 2xy^3\right)dx + \left(2x^3y - 3x^2y^2 - 5y^4\right)dy = 0$ **(15 Marks)**

b. Find the orthogonal trajectories of the family $\frac{2a}{r} = 1 - \cos\theta$. **(05 Marks)**

6 a. Test for convergence of the series,

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + \sqrt{n+1}}$. **(07 Marks)**

b. Test for convergence of the series,

$\frac{x}{1\cdot 2} + \frac{x^2}{2\cdot 3} + \frac{x^3}{3\cdot 4} + \cdots\cdots + \infty$. **(07 Marks)**

c. Test the following series for convergence and absolute convergence,

$1 - \frac{1}{5} + \frac{1}{9} - \frac{1}{13} + \cdots\cdots$ **(06 Marks)**

7 a. If $\left(l_1, m_1, n_1\right)$ and $\left(l_2, m_2, n_2\right)$ are the direction cosines of two lines subtending an angle θ between them. Then prove that $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$. **(07 Marks)**

b. Find the image of the point (1, -1, 2) in the plane $2x + 2y + z = 1$. **(07 Marks)**

c. Find the magnitude and equations of the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$. **(06 Marks)**

8 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is time. Find the components of velocity and acceleration at time $t = 1$ in the direction of $i - 3j + 2k$. **(07 Marks)**

b. If $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$. Then find $div\vec{F}$ and $curl\vec{F}$. **(07 Marks)**

c. Prove that $\nabla \times \left(\phi \vec{A}\right) = \nabla\phi \times \vec{A} + \phi\left(\nabla \times \vec{A}\right)$. **(06 Marks)**

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First Semester B.E. Degree Examination, June / July 08

Engineering Mathematics - I

Time: 3 hrs. Max. Marks:100

Note : *Answer any FIVE full questions, choosing atleast two from each part.*

PART - A

1 a. Find the n^{th} derivative of $\frac{1}{(x+2)(2x+3)} + e^{2x}\cos x$. (07 Marks)

b. If $y^{1/m} + y^{-1/m} = 2x$ prove that $(x^2-1)y_{n+2} - (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$. (07 Marks)

c. Find the angle between the curves $r = \frac{a}{1+\cos\theta}$, and $r = \frac{b}{1-\cos\theta}$. (06 Marks)

2 a. If $u = \log(x^3+y^3+z^3-3xyz)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$. (07 Marks)

b. If $u = \tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$. (07 Marks)

c. If $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$. Find $J\left(\frac{u\ v\ w}{x\ y\ z}\right)$. (06 Marks)

3 a. Obtain a reduction formula for $I_n = \int \operatorname{cosec}^n x\, dx$. Hence find I_3. (07 Marks)

b. Evaluate $\int_0^\infty \frac{dx}{(1+x^2)^n}$, n>1. (07 Marks)

c. Trace the curve $a^2y^2 = x^2(a^2-x^2)$. (06 Marks)

4 a. Find the length of the curve $y^2 = 4ax$ cutoff by the line $3y = 8x$. (07 Marks)

b. Find the area between the curve $y^2(a+x) = x^2(a-x)$ and the asymptote. (07 Marks)

c. Evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x}dx$, $(\alpha > -1)$ using differentiation under integral sign. (06 Marks)

PART - B

5 a. Solve $\frac{dy}{dx} = \frac{y}{x+\sqrt{xy}}$. (07 Marks)

b. Solve $\frac{x^2dy}{dx} - 2xy - x + 1 = 0; y(1) = 0$. (07 Marks)

c. For the family of curves $x^2 + 3y^2 = cy$ (C – parameter), find the orthogonal family of curves. (06 Marks)

6 a. Find the nature of the series, $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + -----$. (07 Marks)

b. Test for convergence of the series, $\frac{1}{1+x} + \frac{1}{1+2x^2} + \frac{1}{1+3x^3} + ------$. (07 Marks)

c. Test the series for i) Absolute convergence ii) Conditional convergence.

$x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + -----$. (06 Marks)

7 a. Find the angle between any two diagonals of a cube. (07 Marks)

b. Show that the points (0, -1, 0), (2, 1, -1), (1, 1, 1) and (3, 3, 0) are coplanar. (07 Marks)

c. Find the shorter distance between the line $x + y + 2z - 3 = 0 = 2x + 3y + 4z - 4$ and z – axis. (06 Marks)

8 a. A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is time. Find the components of velocity and acceleration at time t = 1 in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. (07 Marks)

b. Find a, b, c, so that the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at (1, 2, -1) has maximum magnitude of 64 in the direction of z – axis. (07 Marks)

c. Prove that $\text{curl}(\phi\vec{F}) = \phi(\nabla\times\vec{F}) + \nabla\phi\times\vec{F}$ (06 Marks)

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First Semester B.E. Degree Examination, Dec.08/Jan.09
Engineering Mathematics - I

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions selecting at least two questions from each part.***
2. ***Answer all objective type questions only in first and second writing pages.***
3. ***Answer for Objective type questions shall not be repeated.***

1 a. i) If $y = x^{2n}$ then y_{n+1} is
A) $\frac{(2n)!}{(n-1)!}x^{n-1}$ B) $\frac{(2n)!}{n!}x^{n-1}$ C) $\frac{(n-1)!}{(2n)!}x^{n-1}$ D) Zero

ii) If two curves intersect orthogonally in Cartesian form, the angle between the same two curves in polar form is,
A) $\pi/4$ B) Zero C) 1 radian D) None of these

iii) If the angle between the radius vector and the tangent is constant, then the curve is,
A) $r = ae^{b\theta}$ B) $r = a\cos\theta$ C) $r^2 = a^2\cos(2\theta)$ D) $r = a\theta$

iv) The n^{th} derivative of a constant function is,
A) n B) 1 C) Zero D) ∞ **(04 Marks)**

b. Find the nth derivative of $\frac{x+3}{(x-1)(x+2)}$. **(04 Marks)**

c. If $y = \sin(m\sin^{-1}x)$ express $(1-x^2)y_{n+2} - (2n+1)xy_{n+1}$ in terms n^{th} derivative of y. **(06 Marks)**

d. Find the pedal equation of the polar curve $r = a(1+\cos\theta)$. **(06 Marks)**

2 a. i) If $u = x^n + y^n$ then $\frac{\partial^n u}{\partial x^{n-1}\partial y}$ is equal to $((n \geq 2)$
A) Zero B) $(n!)x + ny^{n-1}$ C) $(n!)x$ D) $(2n)!$

ii) If $u = \sin(x+ay) + g(x-ay)$ then the value of $\frac{\partial^2 u}{\partial^2 y}$ is
A) $\frac{\partial^2 u}{\partial x^2}$ B) $a\frac{\partial^2 u}{\partial x^2}$ C) $a^2\frac{\partial^2 u}{\partial x^2}$ D) $-a^2\frac{\partial^2 u}{\partial x^2}$

iii) If $u = f(x^2+y^2+z^2)$ and $\frac{\partial u}{\partial x} = 2xf'$ then f' is derivative with respect to
A) x B) y C) z D) $x^2+y^2+z^2$

iv) If u and v are the two functions depending on the independent variables x and y then u and v are independent of each other if and only if, for $J = J\left(\frac{u,v}{x,y}\right)$
A) $J = 0$ B) $J \neq 0$ C) $J = 1$ D) $J = -1$ **(04 Marks)**

b. If $u = x^2y + y^2z + z^2x$ show that $u_x + u_y + u_z = (x+y+z)^2$. **(04 Marks)**

c. If $u = x\log(xy)$ where the implicit relation between x and y is $x^3 + y^3 + 3xy = 1$ find $\frac{du}{dx}$. **(06 Marks)**

d. Define 'relative error' and 'percentage error'. Find the error in calculating the power $\omega = \frac{V^2}{R}$ due to errors h and k respectively in measuring voltage V and reistance R. **(06 Marks)**

3 a. i) The value of $\int_0^{\pi} \sin^4 x dx$ is

A) $\frac{3\pi}{8}$ B) $\frac{3\pi}{16}$ C) $\frac{3\pi^2}{8}$ D) zero

ii) The value of $\int_0^{\pi/2} \sin^{99}(x)\cos(x)dx$ is

A) $\frac{1}{99}$ B) $\frac{\pi}{100}$ C) $\frac{99}{100}$ D) None of these

iii) The tangents to the curve $x^3 + y^3 = 3axy$ at origin are

A) $y = x$ and $y = -x$ B) $x = 0,\ y = 0$

C) Line perpendicular to $y = x$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ D) Do not exist

iv) If the equation of the curve remains unchanged after changing r to –r the curve $r = f(\theta)$ is symmetric about

A) Initial line B) A line perpendicular to initial line through pole

C) Radially symmetric about the point pole D) Symmetry does not exist. **(04 Marks)**

b. Evaluate $I = \int_0^{\pi} x \sin^7 x dx$. **(04 Marks)**

c. Obtain the reduction formula for $\int \tan^n x dx$ and hence find the reduction formula for $\int_0^{\pi/4} \tan^n x dx$. **(06 Marks)**

d. Trace the curve $r = a\sin(2\theta)$. **(06 Marks)**

4 a. i) If the derivative of arc length $\frac{ds}{dr} = \phi(r)$ then $\phi(r)$ is

A) $\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$ B) $\sqrt{r^2\left(\frac{d\theta}{dr}\right)^2 + 1}$ C) $\sqrt{r \Big/ \left(\frac{dr}{d\theta}\right)^2}$ D) $\sqrt{s^2 + r^2}$

ii) If S_1 and S_2 are surface areas of the solids generated by revoling the ellipses $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ about the y-axis and then

A) $S_1 > S_2$ B) $S_1 < S_2$ C) $S_1 = S_2$ D) Cant predict

iii) If V_1 = volume of the solid generated by revolving area included between x-axis and $x^2 + y^2 = a^2$ about x-axis

V_2 = volume of the solid generated by the entire area of the circle $x^2 + y^2 = a^2$ about x-axis then

A) $V_1 = V_2$ B) $V_2 = 2V_1$ C) $V_2 = 4V_1$ D) $V_2 = 16V_1$

4 iv) The length of the arc in parametric form is

A) $s=\int_{t_1}^{t_2}\sqrt{1+\left(\frac{dy}{dt}\right)^2}\,dt$ B) $s=\int_{t_1}^{t_2}\sqrt{1+\left(\frac{dx}{dt}\right)^2}\,dt$

C) $s=\int_{t_1}^{t_2}\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$ D) $s=\int_{t_1}^{t_2}\sqrt{(dx)^2+(dy)^2}\,dt$ **(04 Marks)**

b. Find the volume of the solid generated by revolving the part of the parabola $y^2=4ax$ lying between the vertex and the latus-rectum, about the x-axis. **(04 Marks)**

c. Find the surface area of the solid of revolution of the curve $r=2a\cos\theta$ about the initial line. **(06 Marks)**

d. Evaluate $\int_0^1 \frac{x^\alpha-1}{\log x}dx$, $\alpha\ge 0$. **(06 Marks)**

Part B

5 a. i) The order of the differential equation $\sqrt{\frac{dy}{dx}}=(4x+y+1)$ is

A) 1 B) ½ C) zero D) does not exist

ii) The differential equation $\frac{dy}{dx}=\sin(x+y+1)$ with $y(0)=1$ is

A) zero value problem B) Infinite solution problem
C) Initial value problem D) None of these

iii) By Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ in the differential $f\left(x,y,\frac{dy}{dx}\right)=0$ we get the differential equation of,

A) Polar trajectory B) Parametric trajectory
C) Orthogonal trajectory D) Parallel trajectory

iv) In the homogeneous differential equation $\frac{dy}{dx}=\frac{f(x,y)}{\phi(x,y)}$ the degrees of the homogeneous functions f(x, y) and $\phi(x,y)$ are,

A) Same B) Different C) Relatively prime D) Exactly one **(04 Marks)**

b. Solve $e^x\tan y\,dx+(1-e^x)\sec^2 y\,dy=0$. **(04 Marks)**

c. Solve $x\log x\frac{dy}{dx}+y=2\log x$. **(06 Marks)**

d. Find the orthogonal trajectory of $r^2=a^2\cos(2\theta)$. **(06 Marks)**

6 a. i) The sum of infinite series $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+........$ is

A) 9.999... B) 99.999.... C) ∞ D) Indeterminate

ii) If the positive term infinite series $\sum_{n=1}^{\infty}u_n$ and $\sum_{n=1}^{\infty}v_n$ are divergent then $\sum_{n=1}^{\infty}u_n-\sum_{n=1}^{\infty}v_n$ is

A) Convergent B) Divergent C) Oscillatory D) Cant predict

iii) If an arbitrary term infinite series $\sum_{n=1}^{\infty}u_n$ is divergent then its absolute term series $\sum_{n=1}^{\infty}|u_n|$ is,

A) Convergent B) Divergent C) Either convergent or divergent D) Cant predict

6 iv) If $\sum u_n$ is positive term infinite series and if $\lim_{n\to\infty} u_n = 0$ then $\sum u_n$ is

A) Convergent B) Divergent C) Either convergent or divergent D) Oscillatory **(04 Marks)**

b. Test the convergence of the series,

$$\frac{1}{(1)(4)(5)}+\frac{1}{(2)(9)(11)}+\frac{1}{(3)(14)(17)}+\frac{1}{(4)(19)(23)}+\ldots\ldots$$ **(04 Marks)**

c. Test the convergence of $\sum_{n=1}^{\infty}\frac{4.7\ldots\ldots(3n+1)}{1.2\ldots\ldots n}x^n$ **(06 Marks)**

d. Test the absolute and conditional convergence of the following series:

i) $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\ldots..$ ii) $1-\frac{1}{2^3}+\frac{1}{3^3}-\frac{1}{4^3}+\ldots\ldots$ **(06 Marks)**

7 a. i) If l, m, n are direction cosines of a straight line then,

A) $l+m+n=1$ B) $l^2+m^2+n^2=1$ C) $l=m=n$ D) $\frac{l}{m}=\frac{m}{n}=\frac{n}{l}$

ii) Skew lines are,

A) Intersecting B) Parallel C) Planar D) Not coplanar

iii) The angle between the two lines with direction ratios (1, 1, 2) (2, 0, -1) is

A) $0°$ B) $45°$ C) $90°$ D) $\cos^{-1}\frac{3}{5}$

iv) A point on the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z}{-1}$ is

A) (1, 6, 1) B) (1, 6, -1) C) (-1, 6, -1) D) (1, -6, 1) **(04 Marks)**

b. Find the intercept form of a plane $2x+3y+4z+k=0$ passing through a point (1, 1, 1). **(04 Marks)**

c. Find the equation of a plane passing through the line of intersection of the planes $7x-4y+7z+16=0$ and $4x+3y-2z+13=0$ and perpendicular to the plane $x-y-2z+5=0$ **(06 Marks)**

d. Find the magnitude and the equations of the shortest distance between the lines $\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}$ and $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$. **(06 Marks)**

8 a. i) If $\vec{V}=x^2i+y^2j+z^2k$ then $\vec{V}$ at (x, y, z) = (1, 1, 1) becomes

A) Unit vector B) Constant vector C) Scalar D) Complex number

ii) If f is a scalar function then $\nabla f = \text{grad} f$ is

A) Scalar point function B) Vector point function
C) Both A and B D) Neither A nor B.

iii) div curl F is equal to

A) zero B) unity C) $i+j+k$ D) does not exist

iv) If a particle moves along a curve $\vec{R}(t)=x(t)i+y(t)j+z(t)k$ then $\frac{dR}{dt}$ is

A) Radial vector B) Tangential vector C) Normal vector D) Unit vector **(04 Marks)**

b. Find a unit vector normal to the surface $x^3y^3z^2=4$ at the point (-1, -1, 2). **(04 Marks)**

c. Prove that div Curl F = $\nabla\cdot\nabla\times F=0$. **(06 Marks)**

d. If $\vec{V}=3xy^2z^2i+y^3z^2j-2y^2z^3k$ and $\bar{F}=(x^2-yz)i+(y^2-zx)j+(z^2-xy)k$ then prove that $\vec{V}$ is solenoidal and F is irrotational. **(06 Marks)**

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III cope

(5)

USN

06MAT11

First Semester B.E. Degree Examination, June-July 2009
Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: 1. *Answer any Five full questions, choosing at least two from each part.*
2. *Answer all objective type questions only in OMR sheet page 5 of the Answer Booklet.*
3. *Answer to the objective type questions on sheets other than OMR will not be valued.*

PART – A

1 a. i) The n^{th} derivative of $\frac{1}{(ax+b)^2}$ is

(A) $\frac{(-1)^n n! a^n}{(ax+b)^{n+1}}$ (B) $\frac{(-1)^n n+1! a^n}{(ax+b)^{n+2}}$ (C) $\frac{n+1! a^n}{(ax+b)^n}$ (D) $\frac{n! a^n}{(ax+b)^{n+1}}$

ii) If $y^2 = f(x)$, a polynomial of degrees 3, then $2\frac{d}{dx}\left(y^3 \frac{d^2y}{dx^2}\right)$ equals

(A) $f'''(x)+f''(x)$ (B) $f(x)f''(x)$ (C) $f(x)f'''(x)$ (D) $f'''(x)f(x)$

iii) The Pedal equation in polar coordinate system

(A) $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$ (B) $|\phi_1 - \phi_2|$ (C) $\tan\phi - r\frac{d\theta}{dr}$ (D) $\cot\phi = r\frac{dr}{d\theta}$

iv) The curve $r = \frac{a}{1+\cos\theta}$ intersect orthogonally with the following curve

(A) $r = \frac{b}{1-\cos\theta}$ (B) $r = \frac{b}{1-\sin\theta}$ (C) $r = \frac{c}{1+\sin\theta}$ (D) $r = \frac{d}{1+\cos^2\theta}$ **(04 Marks)**

b. Find the nth derivative of y = coshx sinx **(04 Marks)**

c. If $y = \left[x + \sqrt{x^2+1}\right]^m$ prove that $(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$ **(06 Marks)**

d. Show that the pairs of curves $r = a(1+\cos\theta)$ & $r = b(1-\cos\theta)$ intersect orthogonally. **(06 Marks)**

2 a. i) If $f(x,y) = \frac{1}{x^3} + \frac{1}{y^3} + \frac{1}{x^3+y^3}$, then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is

(A) 0 (B) 3f (C) 9 (D) – 3f

ii) If u = f(x-y, y-z, z-x), then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

(A) 2 (B) 0 (C) 1 (D) x + y + z

iii) If an error of 1% is made in measuring its base and height, the percentage error in the area of a triangle is

(A) 0.2% (B) 1% (C) 2% (D) 0.1%

iv) In polar coordinates, $x = r\cos\theta$, $y = r\sin\theta$ then $\partial(x,y)/\partial(r,\theta)$ is equal to

(A) r^3 (B) r^2 (C) r (D) - r **(04 Marks)**

b. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$. **(04 Marks)**

c. If $u = x^2 - y^2$, $v = 2xy$ and $x = r\cos\theta$, $y = r\sin\theta$ then determine the Jacobian $\frac{\partial(u,v)}{\partial(r,\theta)}$. **(06 Marks)**

d. Two sides of a triangle are 10cm & 12cm respectively, the angle between them is measured as 15° with an error of 15 mins. Find the error in the calculated length of the third side of the triangle due to error in the angle. **(06 Marks)**

3 a. i) The value of the definite integral $\int_{-1}^{+1} |x|\,dx$ is equal to

(A) 0 (B) 1 (C) $\pi/2$ (D) $\pi/4$

ii) The asymptote for the curve $x^3 + y^3 = 3axy$ is equal to

(A) $x + y + a = 0$ (B) $x - y - a = 0$ (C) No asymptotes (D) $x + y - a = 0$

iii) If $I_n = \int_0^{\pi/4} \cot^n \theta\, d\theta$, then $n(I_{n-1} + I_{n+1})$ is equal to

(A) 0 (B) 1 (C) 3 (D) None of these.

iv) The value of the definite integral $\int_0^{\infty} \frac{x^2}{(1+x^2)^{7/2}}\,dx$ is equal to

(A) 4/15 (B) $2\pi/15$ (C) 2/15 (D) 15/2 **(04 Marks)**

b. Obtain the reduction formula for $\int \tan^n x\, dx$. **(04 Marks)**

c. Evaluate $\int_0^{\pi} x \sin^2 x \cos^4 x\, dx$. **(06 Marks)**

d. Trace the curve $y^2(a-x) = x^3$, $a > 0$. **(06 Marks)**

4 a. i) The volume generated by the parabola $y^2 = 4ax$ when revolved about the y-axis between $y = 0$ & $y = 2a$ is

(A) $\frac{2\pi a^3}{5}$ (B) $\frac{32\pi a^5}{5a^2}$ (C) $\frac{5\pi a^2}{3}$ (D) $\frac{10\pi^2 a^3}{5}$

ii) The entire length of the cardioid $r = 5(1 + \cos\theta)$ is

(A) 40 (B) 30 (C) 20 (D) 5

iii) If $x = x(t)$, $y = y(t)$ then ds/dt is equal to

(A) $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ (B) $\sqrt{\left(\frac{dx}{dt}\right)^2 - \left(\frac{dy}{dt}\right)^2}$ (C) $\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ (D) None of these

iv) $\frac{d}{d\alpha}\left[\int_a^b f(x,\alpha)\,dx\right]$ is equal to

(A) $\int_a^b \frac{d}{d\alpha} f(x,\alpha)\,dx$ (B) $\int_a^b \frac{\partial}{\partial\alpha} f(x,\alpha)\,dx$ (C) $\int_b^a \frac{\partial}{\partial\alpha} f(x,\alpha)\,dx$ (D) 0 **(04 Marks)**

b. Find $ds/d\theta$ and ds/dr for the curve $r = a(1 - \cos\theta)$. **(04 Marks)**

c. Find the surface area of the solid generated by revolving the cycloid $x = a(t + \sin t)$ $y = a(1 + \cos t)$ **(06 Marks)**

d. Given that $\int_0^{\pi} \frac{dx}{\alpha - \cos x} = \frac{\pi}{\sqrt{\alpha^2 - 1}}$, hence evaluate $\int_0^{\pi} \frac{dx}{(\alpha - \cos x)^2}$ **(06 Marks)**

PART – B

5 a. i) The solution of the differential equation $\frac{dy}{dx} = xe^{y-x^2}$

(A) $2e^{-y} + e^{-x^2} = c$ (B) $e^{-y} - e^{-x^2} = c$ (C) $e^{y-x^2} = c$ (D) $e^{y+x^2} - c = 0$

ii) The integrating factor of the differential equation $\frac{dx}{dy} + \frac{3x}{y} = \frac{1}{y^2}$

(A) e^{y^3} (B) y^3 (C) x^3 (D) $-y^3$

iii) The necessary condition for the differential equation to be exact

(A) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ (B) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ (C) $\frac{\partial M}{\partial y} + \frac{\partial N}{\partial x} = 0$ (D) $\frac{\partial M}{\partial y} = -\frac{\partial N}{\partial x}$

iv) The orthogonal trajectory of $y^2 = 4a(x + a)$ is

(A) $y^2 = 4a(x + a)$ (B) $x^2 = 4a(y + a)$ (C) $y = mx + c$ (D) None of these. **(04 Marks)**

b. Solve $e^y\left(\frac{dy}{dx} + 1\right) = e^x$ **(04 Marks)**

c. Solve $\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}$ **(06 Marks)**

d. Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{a^2 + \lambda} = 1$. **(06 Marks)**

6 a. i) If $\lim_{n \to \infty} \frac{U_{n+1}}{U_n} = l$, then the series is convergent if

(A) $l < 1$ (B) $l > 1$ (C) $l = 1$ (D) $l = 0$

ii) $\sum \frac{1}{n(n+2)}$ series is

(A) Convergent (B) Divergent (C) Oscillatory (D) Absolutely convergent.

iii) Every absolutely convergent series is necessarily

(A) Divergent (B) Convergent (C) Conditionally convergent (D) None of these

iv) The convergence of the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$.......... is tested by

(A) Ratio test (B) Raabe's test (C) Leibnitz test (D) Cauchy Riot test. **(04 Marks)**

b. Examine the series $\frac{1}{1.3.5} + \frac{2}{3.5.7} + \frac{3}{5.7.9}$.......... for convergence. **(04 Marks)**

c. Test the series for convergence $1 + \frac{2}{3}x + \frac{2.3}{3.5}x^2 + \frac{2.3.4}{3.5.7}x^3$.........., $x > 0$. **(06 Marks)**

d. Find the nature of the series $\frac{x}{1.2} - \frac{x^2}{2.3} + \frac{x^3}{3.4} - \frac{x^4}{4.5} +$.........., $x > 0$. **(06 Marks)**

7 a. i) if $2x + 3y + 4z + 5 = 0$ is the equation of a plane, then 2, 3, 4 represent

(A) Direction ratios of the normal to the plane

(B) Direction cosines of the normal to the plane

(C) Direction ratios of a line parallel to the plane

(D) None of these

ii) A line makes angles α, β, γ with the co-ordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is equal to

(A) 1 (B) 2 (C) 8/3 (D) 4/3

iii) The length of the perpendicular from the origin onto the plane $3x + 4y + 12z = 52$ is

(A) 4 (B) 3 (C) 0 (D) - 1

iv) The two lines are said to be parallel if

(A) $a_1a_2 + b_1b_2 + c_1c_2 = 0$ (B) $a_1/a_2 = b_1/b_2 = c_1/c_2$

(C) $a_1/b_1 + a_2/b_2 + c_1/c_2 = 0$ (D) None of these. **(04 Marks)**

b. Show that the angle between any two diagonals of a cube is $\cos^{-1}(1/3)$. **(04 Marks)**

c. Show that the lines $\frac{x+1}{1} = \frac{y+1}{2} = \frac{z+1}{3}$ and $x + 2y + 3z - 8 = 0 = 2x + 3y + 4z - 11$ intersect. Find their point of intersection and the equation of the plane containing them. **(06 Marks)**

d. Find the image of the point $(2, -1, 3)$ in the plane $2x + 4y + z - 24 = 0$. **(06 Marks)**

8 a. i) The velocity of the moving particle along the curve $x = e^{-t}$, $y = 2\cos 3t$, $z = 2\sin 3t$ is

(A) $-e^{-t}i - 6\sin 3tj + 6\cos 3tk$ (B) $e^{-t}i - 18\cos 3tj - 18\sin 3tk$

(C) $e^{-t}i + 2\cos 3tj + 2\sin 3tk$ (D) $e^{-t} - 6\sin 3t$

ii) The resultant of a gradient is
(A) Vector (B) Scalar (C) Irrotational (D) Field

iii) If the vector $\vec{F} = (x+3y)i + (y-2z)j + (x+az)k$ is Solenoidal then a is equal to
(A) 2 (B) - 2 (C) 0 (D) 1

iv) If $F = x^2 + y^2 + z^2$, then curl grad F is
(A) 1 (B) 0 (C) - 1 (D) 2 **(04 Marks)**

b. Find the angle between the surfaces $\phi = x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2,–1, 2). **(04 Marks)**

c. Show that $\vec{F} = \dfrac{xi + yj}{x^2 + y^2}$ is both Solenoidal & irrotational. **(06 Marks)**

d. Prove that curl curl $\vec{F}$ = grad div $\vec{F} - \nabla^2 \vec{F}$ **(06 Marks)**

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06MAT11

First Semester B.E. Degree Examination, Dec.09/Jan.10
Engineering Mathematics - I

Time: 3 hrs. Max. Marks:100

Note: *1.Answer any FIVE full questions, choosing at least two from each part.*
2.Answer all objective type questions only in OMR sheet page 5 of the Answer Booklet.
3.Answer to objective type questions on sheets other than OMR will not be valued.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. i) The n^{th} derivative of sinh ax is

A) $\frac{a^n}{2}\left[e^{ax} - (-1)^n e^{-ax}\right]$ B) $\frac{a^n}{2}\left[e^{ax} + (-1)^n e^{-ax}\right]$

C) $\frac{a^n}{2}\left[e^{-ax} + (-1)^n e^{ax}\right]$ D) $\frac{a^n}{2}\left[e^{-ax} - (-1)^n e^{ax}\right]$

ii) The angle between radius vector and the tangent to the curve $r = ae^{\theta\cot\alpha}$ at any point is

A) $\pi/2$ B) α C) 0 D) $\pi/4$

iii) The angle between the curves $r = 2\sin\theta$ and $r = \sin\theta + \cos\theta$ is

A) $\pi/2$ B) 0 C) $\pi/4$ D) $\pi/8$

iv) Pedal equation to the curve $r = a(1 + \cos\theta)$ is

A) $r^2 = 2ap^3$ B) $r = 3ap$ C) $r^3 = 2ap$ D) $r^3 = 2ap^2$ **(04 Marks)**

b. Find the nth derivative of $\log(4x^2 - 1)$. **(04 Marks)**

c. If $y = \frac{\sinh^{-1} x}{\sqrt{1+x^2}}$, prove that $(1+x^2)y_{n+2} + (2n+3)xy_{n+1} + (n+1)^2y_n = 0$ **(06 Marks)**

d. Find the pedal equation to the curve $r^n = a^n\cos n\theta$ **(06 Marks)**

2 a. i) If $u = (x - y)^4 + (y - z)^4 + (z - x)^4$ then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ is

A) 1 B) u C) 4u D) 0

ii) If $u = f(x + ay) + g(x - ay)$ then $\frac{\partial^2 u}{\partial y^2}$ is

A) $\frac{\partial^2 u}{\partial x^2}$ B) $a\frac{\partial^2 u}{\partial x^2}$ C) $a^2\frac{\partial^2 u}{\partial x^2}$ D) $\frac{\partial^2 u}{\partial x \partial y}$

iii) If $u = \cos^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$ then $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy}$ is

A) u B) 2u C) 0 D) 1

iv) If $x = uv$ and $y = \frac{u}{v}$ then $\frac{\partial(x,y)}{\partial(u,v)}$ is

A) $-\frac{2u}{v}$ B) $-\frac{2v}{u}$ C) 0 D) 1 **(04 Marks)**

b. If u is a homogeneous function of degree n prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$. **(04 Marks)**

c. If $u = f(x - y, y - z, z - x)$ prove that $u_x + u_y + u_z = 0$. **(06 Marks)**

d. If $x = r\cos\theta$, $y = r\sin\theta$ and $J = \frac{\partial(x,y)}{\partial(r,\theta)}$, $J' = \frac{\partial(r,\theta)}{\partial(x,y)}$ show that $JJ' = 1$. **(06 Marks)**

3 a. i) The value of $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right)dx$ is

A) $\frac{16}{15}$ B) $\frac{2}{15}$ C) 1 D) $\frac{15}{16}$

ii) The asymptote to the curve $x^3 + y^3 = 3axy$ is

A) $x + y = 0$ B) $x + y - a = 0$ C) $x + y + a = 0$ D) $x - y = 0$

iii) The value of $\int_0^1 x^6\sqrt{1-x^2}\, dx$ is

A) $\frac{8\pi}{135}$ B) $\frac{\pi}{16}$ C) $\frac{5\pi}{256}$ D) $\frac{5\pi}{126}$

iv) If n is odd, the value of $\int_0^{\pi} \sin^m x \cos^n x\, dx$ is

A) 1 B) 2 C) 3 D) 0 **(04 Marks)**

b. Obtain the reduction formula for $\int \sin^n x\, dx$. **(04 Marks)**

c. Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\, dx$ **(06 Marks)**

d. Trace the curve $y^2(a - x) = x^2(a + x)$, $a > 0$. **(06 Marks)**

4 a. i) The length of the arc of the curve $y = \log(\sec x)$ between the points with $x = 0$ and $x = \pi/3$ is

A) $\log(2 + \sqrt{3})$ B) $\log(2 - \sqrt{3})$ C) $\log(\sqrt{3} + 2)$ D) $\log(\sqrt{3} - 2)$

ii) The area bounded by the parabola $y = 4x - x^2$ and the line $y = x$ is

A) 1 B) 1/2 C) 9/2 D) 2/9

iii) The surface area generated when the curve $y = f(x)$, $a \le x \le b$ is revolved about x axis is

A) $\int_a^b \pi y^2 ds$ B) $\int_a^b 2\pi y ds$ C) $\int_b^a y dx$ D) $\int_a^b y^2 dx$

iv) The volume generated when the curve $y = \frac{x}{1+x^2}$, $0 \le x \le \infty$ is revolved about x-axis is

A) $\frac{\pi^2}{4}$ B) $\frac{\pi}{4}$ C) π D) 2π **(04 Marks)**

b. Find the perimeter of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$. **(04 Marks)**

c. Find the volume generated when the curve $r = a(1 + \cos\theta)$ is revolved about initial line. **(06 Marks)**

d. Using the differentiation under integral sign evaluate $\int_0^1 \frac{x^{\alpha} - 1}{\log x} dx$, $\alpha \ge 0$. **(06 Marks)**

PART – B

5 a. i) The solution of the differential equation $(x^2 - 3y^2)dy = 2xy\,dx$ is
A) $x^2 = 3y^2 + Cy$ B) $x^2 + 3y^2 = Cy$ C) $3x^2 + y^2 = Cx$ D) $x^2 + 3y^2 = Cx$

ii) The solution of the differential equation $\frac{dy}{dx} + y\cot x = \cos x$ is
A) $2y = \operatorname{cosec} x + A \sin x$ B) $y = A \sin x + \operatorname{cosec} x$
C) $2y = \sin x + A \operatorname{cosec} x$ D) $y = \sin x + A \operatorname{cosec} x$

iii) The integrating factor for the differential equation $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$ is
A) $\frac{1}{x+1}$ B) $\frac{1}{(x+1)^2}$ C) $\log(x+1)$ D) $\log x$

iv) The orthogonal trajectory of the family $x^2 + y^2 = c^2$ is
A) $x + y = c$ B) $xy = c$ C) $x^2 + y^2 = x + y$ D) $y = cx$ **(04 Marks)**

b. Solve $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$. **(04 Marks)**

c. Solve $x^2y\,dx - (x^3 + y^3)\,dy = 0$. **(06 Marks)**

d. Find the orthogonal trajectory of the family $r^n \cos n\theta = a^n$. **(06 Marks)**

6 a. i) If $\sum_1^\infty u_n$ is convergent series of positive terms then $\lim_{n\to\infty} u_n$ is
A) 1 B) ± 1 C) 0 D) > 0

ii) The series $\sum_1^\infty \sqrt{n^2+1} - 1$ is
A) Convergent B) Divergent C) Oscillatory D) None of these.

iii) $\sum_1^\infty \frac{x^n}{n(n+1)}$ converges if
A) $x \le 1$ B) $x \ge 1$ C) $x > 1$ D) All x

iv) The series $\sum \frac{x^n}{(n+1)^n}$, $x > 0$ is
A) Divergent B) Convergent C) Oscillatory D) None of these. **(04 Marks)**

b. Test the convergence of the series $\frac{1}{4.7.10} + \frac{4}{7.10.13} + \frac{9}{10.13.16} + \ldots\ldots\ldots$ **(04 Marks)**

c. Test the convergence of the series $\sum_1^\infty \left(1 - \frac{3}{n}\right)^{n^2}$ **(06 Marks)**

d. Test the series $\frac{x}{\sqrt{3}} - \frac{x^2}{\sqrt{5}} + \frac{x^3}{\sqrt{7}} - \ldots\ldots$ for absolute convergence and conditional convergence. **(06 Marks)**

7 a. i) The angle between the line $\frac{x-1}{2}=\frac{y-1}{3}=\frac{z-1}{6}$ and XOY plane is

A) $\sin^{-1}\left(\frac{7}{6}\right)$ B) $\sin^{-1}\left(\frac{6}{7}\right)$ C) $\cos^{-1}\left(\frac{6}{7}\right)$ D) $\cos^{-1}\left(\frac{7}{6}\right)$

ii) The equation of the plane passing through (4, –2, 1) and perpendicular to the line with direction cosines 7, 2, –3 is

A) $x + 3y - 4z - 8 = 0$ B) $2x + 7y - 3z - 24 = 0$

C) $7x + 2y - 3z - 21 = 0$ D) $7x + 3y - 2z + 21 = 0$

iii) If the lines $\frac{x-3}{1}=\frac{y-2}{3}=\frac{z-1}{4}$ and $\frac{x-4}{2}=\frac{y-2}{3}=\frac{z+6}{k}$ are coplanar then 'k' is

A) – 2 B) 3 C) +2 D) – 3

iv) The lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ and $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ intersect at

A) (5, –7, 6) B) (7, 5, –6) C) (5, –6, 7) D) (7, 6, –5) **(04 Ma**

b. Find the foot of the perpendicular from (1, 1, 1) to the line joining the points (1, 4, 6) and (5, 4, 4) **(04 Marks)**

c. Find the equation of the plane passing through the point (-1, 2, 1), (-3, 2, -3) and parallel to Y-axis. **(06 Marks)**

d. Find the point of intersection of the lines

$\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ and $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ **(06 Marks)**

8 a. i) A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$, $z = 2t - 5$. The acceleration at $t = 1$ is

A) 6i – 2j B) -6i + 2j C) 2i – 6j D) 2i + 6j

ii) The unit normal vector to the surface $x^2y + 2xz = 4$ at the point (2, –2, 3) is along

A) i – 2j – 2k B) i + 2j – k C) 2i + j + k D) i – j – 2k

iii) If $\vec{F}=(x + y + 1)i + j - (x + y)k$ then $\vec{F}\cdot \text{curl}\vec{F}$ is

A) 1 B) – 1 C) 0 D) 2

iv) If $\phi = 2x^3y^2z^4$ then $\nabla^2\phi$ at (1, 1, 1) is

A) 20 B) 0 C) 10 D) 40 **(04 Marks)**

b. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2, -1, 2) **(04 Marks)**

c. Show that $\vec{F}=(2xy^2 + yz)i + (2x^2y + xz + 2yz^2)j + (2yz^2 + xy)k$ is a conservative force field and find its scalar potential. **(06 Marks)**

d. If ϕ is a scalar field and $\vec{F}$ is a vector field prove that $\nabla.Q\vec{F} = \phi(\nabla.\vec{F}) + \nabla\phi.\vec{F}$ **(06 Marks)**

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06MAT11

First Semester B.E. Degree Examination, May/June 2010
Engineering Mathematics - I

Time: 3 hrs. Max. Marks:100

Note: 1. *Answer any FIVE full questions, choosing at least two from each part.*
2. *Answer all objective type questions only on OMR sheet page 5 of the Answer Booklet.*
3. *Answer to objective type questions on sheets other than OMR will not be valued.*

PART – A

1 a. i) The n^{th} derivative of $\frac{1}{x^p}$ is

A) $\frac{(-1)^{n+1}(p+n)!}{(p-1)!\,x^{p+n}}$ B) $\frac{(-1)^{n+1}(p+n-1)!}{(p-1)!\,x^{p+n}}$ C) $\frac{(-1)^{n}(p+n-1)!}{(p-1)!\,x^{p+n}}$ D) $\frac{(-1)^{n}(p+n-1)!}{p!\,x^{p}}$

ii) The n^{th} derivative of e^x is

A) $a^n e^{ax}$ B) ae^x C) a^2e^x D) e^x

iii) The angle between radius vector and tangent is

A) $\tan\phi = r\frac{d\theta}{dr}$ B) $\tan\phi = r^2\frac{d\theta}{dr}$ C) $\tan\phi = \frac{1}{r}\frac{d\theta}{dr}$ D) $\tan\phi = \frac{dr}{d\theta}$

iv) The curve $r = \frac{a}{1+\cos\theta}$ intersect orthogonally with the following curve:

A) $r = \frac{b}{1-\cos\theta}$ B) $r = \frac{b}{1+\sin\theta}$ C) $r = \frac{b}{1+\sin^2\theta}$ D) $r = \frac{b}{1+\cos^2\theta}$ **(04 Marks)**

b. Find the n^{th} derivation of $y = \sinh 2x \sin 4x$. **(04 Marks)**

c. If $y = \sinh\left(m\log(x+\sqrt{x^2+1})\right)$, prove that $(x^2+1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$. **(06 Marks)**

d. Find the pedal equation of the curve $r^m = a^m(\cos m\theta + \sin m\theta)$. **(06 Marks)**

2 a. i) If $u = \log\left(\frac{x^2}{y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

A) 2u B) 3u C) u D) 1

ii) If $u = x^3 + y^3$, then $\frac{\partial^3 u}{\partial x^2 \partial y}$ is equal to

A) -3 B) 3 C) 0 D) $3x+3y$

iii) If $x = r\cos\theta$, $y = r\sin\theta$, then $\frac{\partial(x,y)}{\partial(r,\theta)}$ is equal to

A) 1 B) r C) $\frac{1}{r}$ D) 0

iv) If an error of 1% is made in measuring its length and breadth, the percentage error in the area of a rectangle is

A) 0.2% B) 0.02% C) 2% D) 1% **(04 Marks)**

b. If $z = e^{ax+by} + (ax-by)$, prove that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$. **(04 Marks)**

2 c. If $w = f(x, y)$, $x = r\cos\theta$, $y= r\sin\theta$, show that $\left(\frac{\partial t}{\partial x}\right)^2 + \left(\frac{\partial t}{\partial y}\right)^2 = \left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2$. **(06 Marks)**

d. If u, v are functions of r, s and r, s are functions of x, y, prove that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{\partial(u,v)}{\partial(r,s)} \times \frac{\partial(r,s)}{\partial(x,y)}$. **(06 Marks)**

3 a. i) The value of $\int_0^{\pi} \sin^5\left(\frac{x}{2}\right) dx$ is

A) $\frac{16}{25}\pi$ B) $\frac{25}{16}\pi$ C) $\frac{16\pi^2}{25}$ D) $\frac{25}{16}\pi^2$

ii) The curve $y^2(a - x) = x^2(a + x)$ is symmetrical about the _______ axis.

A) x B) y C) both x and y D) none of these

iii) The value of $\int_0^1 x^{3/2}(1-x)^{3/2} dx$ is

A) $\frac{\pi}{32}$ B) $\frac{-\pi}{32}$ C) $\frac{3\pi}{128}$ D) $\frac{-3\pi}{128}$

iv) If $f(r, \theta) = f(-r, \theta)$ then the curve is symmetrical about the ______

A) initial line B) pole C) origin D) tangential line **(04 Marks)**

b. Evaluate $\int_0^{\infty} \frac{x^2}{(1+x^2)^{7/2}} dx$. **(04 Marks)**

c. Obtain the reduction formula for $\int_0^{\pi/4} \text{see}^n x\, dx$. **(06 Marks)**

d. Trace the curve $y^2(a^2 + x^2) = x^2(a^2 - x^2)$. **(06 Marks)**

4 a. i) If $y = f(x)$ be the equation of the Cartesian curve then $\frac{ds}{dx}$ is equal to

A) $\sqrt{1+y_1^2}$ B) $\sqrt{1+y_1}$ C) $-\sqrt{1+y_1^2}$ D) $-\sqrt{1+y^2}$

ii) The area of the cardioid $r = a(1+\cos\theta)$ is

A) $\frac{3}{2}\pi a$ B) $\frac{2}{3}\pi a$ C) $\frac{3}{2}\pi a^2$ D) $\frac{2}{3}\pi a^2$

iii) The surface area of the solid got by revolving the circle $r = 2a\cos\theta$ about the initial line is

A) $4\pi^2 a$ B) $4\pi a^3$ C) $4\pi a^2$ D) $8\pi a$

iv) The volume generated by the revolution of the curve $y = \frac{a^3}{a^2 + x^2}$ about its asymptote is

A) $\frac{\pi^2 a^3}{2}$ B) $\frac{\pi a^3}{2}$ C) $\frac{\pi a^2}{2}$ D) $\frac{\pi a}{2}$ **(04 Marks)**

b. Find the length of the arc of the curve $y = \log \sec x$ between the points for which $x = 0$ and $x = \pi/3$. **(04 Marks)**

4 c. Find the surface area of the solid got by revolving the arch of the cycloid $x = a(t + \sin t)$, $y = a(t + \cos t)$ about the base. **(06 Marks)**

d. Evaluate $\int_0^{\infty} \frac{\tan^{-1} \alpha x}{x(1+x^2)} dx$ where $\alpha \geq 0$ using the rule of differentiation under the integral sign. **(06 Marks)**

PART – B

5 a. i) The order of the differential equation $\left(\frac{dy}{dx}\right)^2 - 5\frac{dy}{dx} + 4y = 0$ is

A) 2 B) 0 C) 3 D) 1

ii) The integrating factor of the differential equation $\frac{dy}{dx} + y\cos x = \frac{\sin 2x}{2}$ is

A) $e^{\sin^2 x}$ B) $e^{\sin^3 x}$ C) $e^{\sin x}$ D) sin x

iii) The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} - \operatorname{cosec}\frac{y}{x}$ is

A) $\cos\left(\frac{y}{x}\right) - \log x = c$ B) $\cos\left(\frac{y}{x}\right) + \log x = c$

C) $\cos^2\left(\frac{y}{x}\right) + \log x = c$ D) $\cos^2\left(\frac{y}{x}\right) - \log x = c$

iv) By replacing $\frac{dr}{d\theta}$ by $-r^2\frac{dr}{d\theta}$ in the differential equation $f\left(r, \theta, -r^2\frac{dr}{d\theta}\right) = 0$, we get the differential equation of________.

A) Orthogonal trajectory B) Polar trajectory
C) Parametric trajectory D) None of these. **(04 Marks)**

b. Solve : $(1 - x^2)\frac{dy}{dx} - xy = 1$. **(04 Marks)**

c. Solve : $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$. **(06 Marks)**

d. Find the orthogonal trajectories of the family of curves $r = 2a(\cos\theta + \sin\theta)$ where a is a parameter. **(06 Marks)**

6 a. i) The series $\frac{1}{1^P} + \frac{1}{2^P} + \frac{1}{3^P} + \ldots\ldots\ldots$ converges if

A) $p > 0$ B) $p < 1$ C) $p > 1$ D) $p \leq 1$

ii) $\sum \sin\left(\frac{1}{n}\right)$ is

A) convergent B) divergent C) oscillatory D) none of these

iii) The convergence of the series $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{5}} + \ldots\ldots\ldots$

A) Leibnitz test B) Raabe's test C) Ratio test D) Cauchy's root test

iv) If a series Σy_n is such that S_n does not tend to unique limit as $n \to \infty$, we say that the series Σy_n is

A) convergent B) divergent C) oscillatory D) none of these **(04 Marks)**

b. Determine the nature of the series $\sum\left(\sqrt{n^2+1} - n\right)$. **(04 Marks)**

6 c. Test the convergence of the series $\frac{2}{3.4}+\frac{2.4}{3.5.6}+\frac{2.4.6}{3.5.7.8}+\ldots\ldots\ldots\ldots$ **(06 Marks)**

d. Find the nature of the series $1-\frac{1}{5}+\frac{1}{9}-\frac{1}{13}+\ldots\ldots$ **(06 Marks)**

7 a. i) A line makes angles α, β, γ with coordinate axes, then $\cos 2\alpha+\cos 2\beta+\cos 2\gamma$ is equal to

A) 1 B) 2 C) −1 D) −2

ii) Find the angle between the planes $x-y+2z-9=0$ and $2x+y+z=7$ is

A) 30° B) 90° C) 60° D) 120°

iii) Two straight lines which lie in the same plane are called

A) parallel B) perpendicular C) coplanar D) non-coplanar

iv) The normal form of plane equation is

A) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ B) $l^2+m^2+n^2=1$

C) $l_1l_2+m_1m_2+n_1n_2=0$ D) $lx+my+nz=p$ **(04 Marks)**

b. Prove that the sum of the squares of the direction cosines of a line is equal to unity. **(04 Marks)**

c. Find the image of the point (1, 2, 3) in the plane $x+y+x=9$. **(06 Marks)**

d. Find the shortest and the equation of the line of shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and the y – axis. **(06 Marks)**

8 a. i) The acceleration of the moving particle along the curve $x=\cos 3t$, $y=\sin 3t$, $z=-t$ is

A) $-3\sin t\,\hat{i}+3\cos 3t\,\hat{j}-\hat{k}$ B) $\cos t\,\hat{i}+\sin 3t\,\hat{j}-\hat{k}$

C) $-9\cos 3t\,\hat{i}-9\sin 3t\,\hat{j}$ D) $-12\cos 3t\,\hat{i}-12\sin 3t\,\hat{j}$

ii) The directional derivative of x^2yz+xz^2 at (-1, 2, 1) in the direction of $2\hat{i}-\hat{j}-2\hat{k}$ is

A) $-\frac{7}{3}$ B) $\frac{7}{3}$ C) $\frac{3}{7}$ D) $-\frac{3}{7}$

iii) If a particle moves along a curve $\vec{R}(t)=x(t)i+y(t)j+z(t)k$ then $\frac{dR}{dt}$ is

A) Radial vector B) Tangential vector C) Normal vector D) Unit vector

iv) Curl (grad ϕ) is equal to

A) unity B) $\hat{i}+\hat{j}+\hat{k}$ C) zero D) none of these **(04 Marks)**

b. Find the angle between the tangents $\vec{r}=t^2\,\hat{i}+2t\,\hat{j}-t^3\,\hat{k}$ at the points $t=\pm 1$. **(04 Marks)**

c. If $\vec{r}=x\,\hat{i}+y\,\hat{j}+z\,\hat{k}$ and $|\vec{r}|=r$, find grad $\left(\text{div}\frac{\vec{r}}{r}\right)$. **(06 Marks)**

d. If $\vec{a}$ is a constant vector and $\vec{r}=x\,\hat{i}+y\,\hat{j}+z\,\hat{k}$, show that $\frac{1}{2}\text{curl}(\vec{a}\times\vec{r})=\vec{a}$. **(06 Marks)**

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